MODEL TEST PAPER

MATHEMATICS FOR CLASS –XII

Note : (i) All questions are compulsory.

(ii) Q. 1 will consist of 10 parts and each part will carry 1 mark.

(iii) Q. 2 to Q. 9 each will be of 2 marks.

(iv) Q. 10 to Q. 19 each will be of 4 marks.

(v) Q. 20 to Q. 23 each will be of 6 marks.

(vi) Q. 10, 15, 18, 21, 22 and 23 contain internal choice.

(vii) Question number 10, 15, 18, 20, 21, 22 and 23 contain internal choice.

1. (i) Let * be a binary operation on set R of real numbers defined by a * b = a + b where a, b ∈ R, then the value of 6 * (3 * 4) is

(a) 15 (b) 12 (c) 13 (d) 32 — 1

(ii) If A + B = C where B and C are matrices of order 3 × 3 then the order of matric C is

(a) 3 × 5 (b) 3 × 3 (c) 5 × 5 (d) 5 × 3 — 1

(iii) Principal value of $\cos^{-1}\left(-\cos\frac{3\pi}{5}\right)$ is

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{5}$ (d) $\frac{2\pi}{5}$ — 1

(iv) $\frac{d}{dx}\{cos^{-1}(e^x)\}$ is equal to

(a) $e^x sin^{-1}(e^x)$ (b) $\frac{e^x}{1-e^x}$ (c) $e^x\, cos^{-1}x$ (d) $\frac{-e^x}{\sqrt{1-e^{2x}}}$ — 1

(v) if f (x) = $\begin{cases} kx^2, & x < 3 \\ 3\ , & x \geq 3 \end{cases}$ is continuous at x=3 then value of k is

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{3}{7}$ (d) $\frac{7}{3}$ — 1

(vi) $\int_0^{\pi} sin^3x\, cos^5 x\, dx$ is equal to

(a) 1 (b) −1 (c) 2 (d) 0 — 1

(vii) The integrating factor of differential equation $\frac{dy}{dx} + \frac{2y}{x} = 3x$ is

(a) x^2 (b) $\frac{1}{x}$ (c) $\frac{1}{x^2}$ (d) x — 1

(viii) Magnitude of the vector $\frac{2}{\sqrt{3}}\hat{i} + \frac{2}{\sqrt{3}}\hat{j} + \frac{2}{\sqrt{3}}\hat{k}$ *is equal to*

(a) -1 (b) 2 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$ 1

(ix) The angle between the lines whose direction ratios are <3, 4, -3> and <-2, 3, 2 > is given by

(a) 45^0 (b) 60^0 (c) 90^0 (d) 30^0 1

(x) If A and B are independent events and if P(A) = $\frac{1}{2}$, P (B)= $\frac{2}{5}$, then P(A∩B) is equal to

(a) $\frac{1}{5}$ (b) $\frac{25}{3}$ (c) $\frac{1}{3}$ (d) $\frac{3}{25}$ 1

2. If x $\begin{bmatrix}3\\4\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\11\end{bmatrix}$, then find the value of x and y. 2

3. If y = e^x tan x, then find $\frac{d^2y}{dx^2}$ 2

4. Evaluate $\int \frac{dx}{1+\sin x}$ 2

5. Evaluate $\int e^x \left(cos^{-1}x - \frac{1}{\sqrt{1-x^2}}\right) dx$ 2

6. Solve the differential equation

$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ 2

7. Solve the differential equation

$(sin^2x + 2\ sinx + 3)\ \frac{dy}{dx} = 2\ (\sin x + 1)\cos x$ 2

8. Find the coordinates of that point when the line passing through two points (1, 2, 3) and (3, 4, 5) crosses XY plane. 2

9. The probability of A and B achieving a target is $\frac{3}{4}$ and $\frac{5}{6}$ respectively. If both of them try then find the probability that at least one of them will achieve the target. 2

10. Check whether the function defined by f: N → N, f(x) = x^2 is one-one and onto. 4

Or

Prove that the relation R defined on the set Z of integers as

R=$\{(a,b): 3\ divides\ |a-b|\}$ is an equivalence relation.

11. Prove that $sin^{-1}\frac{3}{5} + cos^{-1}\frac{15}{17} = sin^{-1}\frac{77}{85}$ 4

12. Express $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrices. 4

13. If y= $(x \sin x)^x$ then find $\frac{dy}{dx}$ 4

14. Using differentials find the approximate value of $\sqrt[3]{0.065}$ 4

15. Evaluate $\int \frac{dx}{x(x^3+1)}$ 4

or

Evaluate $\int_3^5 3^x$ dx as the limit of a Sum. 4

16. Find the area of the region enclosed between the parabolas $x^2 = 4y$ and $y^2 = 4x$. Also draw its rough sketch. 4

17. Find the particular solution of the differential equation

$\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0, (x \neq 0)$, given that y = 0, x = 1 4

18. The scalar product of the vector $\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$ with a unit vector along the sum of vectors $\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\lambda\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ is equal to one. Find the value of λ. 4

or

Using scalar triple product, prove that the four points whose position vectors are given by the vectors $\hat{\imath} - \hat{\jmath} + \hat{k}$, $2\hat{\imath} - 2\hat{\jmath}$, $3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $4\hat{\imath} + \hat{\jmath} + \hat{k}$, are coplanar. 4

19. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The respective probabilities that the Scooter, car and truck will be involved in an accident are $\frac{1}{100}$, $\frac{3}{100}$ and $\frac{3}{20}$. One of the insured people meets with an accident. What is the probability that he is a truck driver? 4

20. Solve the following system of linear equations by matrix method:

$x - y + 2z = 7,\ 3x + 4y - 5z = -5,\ 2x - y + 3\,z = 12$ 6

Or

Using elementary transformation find the inverse of 6

$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & -2 \\ -1 & 1 & 2 \end{bmatrix}$$

21. Find the height of a right circular cylinder of maximum volume, which can be inscribed in a sphere of radius 9 cm. 6

Or

A wire of length 36 cm is to be cut into two pieces, one of the pieces is made into a circle and other into an equilateral triangle. What should be the lengths of the two pieces so that the combined area of both circle and an equilateral triangle is minimum. 6

22. Find the shortest distance between the lines given by the equations:

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ and 6

$\vec{r} = (2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + \mu(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$ and

Or

Find the equation of the plane through the line of intersection of the planes given by the equations $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane given by the equation $x - y + z = 0$ 6

23. Solve the following linear programming problem graphically. Minimize the objective function Z = 2x +3y subject to the constraints 6

$x + y \le 100, x + y \ge 60,\ x \le 60,\ y \le 50,\ x \ge 0, y \ge 0,$

or

Graphically maximize the objective function Z = x+2y subject to constraints.

$x + 2y \ge 100, 2x - y \le 0,\ 2x + y \le 200,\ x \ge 0,\ y \ge 0$ 6