Model Test Paper
Mathematics-XII
Semester-II

Max Time :3 Hours
Max.Marks:100

Note : All questions are compulsory.

1. Q.1 will consist of ten parts. Out of these, four parts will be from the syllabus of Ist semester. Each question will carry 1-mark.
2. Q.2 to Q.16 will carry four marks each.
3. Q.17 to Q.21 i.e. five question each will be of six marks.
4. Use of calculator is not allowed.

Q.1 (I) The value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ is

(a) $\frac{2\pi}{5}$ (b) $\frac{3\pi}{5}$ (c) $\frac{4\pi}{5}$ (d) $\frac{\pi}{5}$

(II) $\frac{d}{dx}\left[\sec^{-1}x + \text{cosec}^{-1}x\right] = ...$

(a) -1 (b) 0 (c) 1 (d) 2

(III) The value of determinant

$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix}$$ is equal to

(a) -3 (b) 2 (c) -1 (d) 0

(IV) If R be the relation in the set N given by R = { (a, b) ; a = b - 2, b > 6 } choose the correct answer.

(a) $(2,4) \in R$ (b) $(3,8) \in R$
(c) $(6,8) \in R$ (d) $(8,7) \in R$

(V) $\int \text{cosec}^2 x \, dx$ is equal to

(a) -cotx +c (b) tanx+c
(c) -tanx+c (d) cosecx.cotx+c

(VI) Area in the 1st quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x = 0 and x=2 is

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

(VII) The number of arbitrary constant in the general solution of a differential equations of fourth order are

(a) 0 (b) 2 (c) 3 (d) 4

(VIII) The angle between the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

(IX) If a line makes angles 90^0,135^0, 45^0, with X, Y and Z axes respectively, its direction cosines are

(a) 1, $\frac{-1}{\sqrt{2}}$, -1 (b) 0 , $\frac{-1}{\sqrt{2}}$, -1

(c) 1, $\frac{-1}{\sqrt{2}}$, 1 (d) 0, $\frac{-1}{\sqrt{2}}$, 1

(X) Two coins are tossed four times. The numbers of elements in sample space are

(a) 8 (b) 4 (c) 16 (d) 36 **[1× 10 = 10]**

Q 2. Find the value of $\int \frac{e^{2x}-1}{e^{2x}+1}$ dx

Q 3. Evaluate $\int e^{x} \left[\frac{1+sinx}{1+cocx}\right]$ dx

Or

Evaluate: $\int \frac{1}{1+cotx}$ dx

Q 4. Find $\int_0^2 (x^2 + 1)\,dx$ as the limit of a sum.

Q 5. Using properties of integrals ,evaluate

$$\int_0^{\frac{\pi}{4}} \log(1 + tanx)\,dx$$

Q 6. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis.

Q 7. Form a differential equation by eliminating arbitrary constants **a** and **b ,** where

$$y = e^x (a \cos x + b \sin x)$$

Q 8. For the differential equation $xy \frac{dy}{dx} = (x+z)(y+z)$.Find the solution curve passing through the point (1 , -1).

Q 9 Show that the given differential equation $(x^2 + xy)\, dy = (x^2 + y^2)\, dx$ is homogeneous and solve it.

Or

Find the general solution of the differential equation

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Q 10. Find the area of the parallelogram whose adjacent sides are determined by the vectors

$$\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k} \text{ and } \vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$$

Q 11. Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k},\ \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k} \text{ and } \vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a} \text{ and } \vec{b}, \text{ and } \vec{c}.\vec{d} = 15$.

Q 12. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

Q 13. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and}$$

$$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Q 14. Find the vector equation of the plane passing through the intersection of the planes

$\vec{r}.(2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$; $\vec{r}.(2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ and through the point (2,1,3)

Q 15. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?

Q 16. A bag contains 4 red and 4 black balls; another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the bag.

Or

Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces.

[4× 15 = 60]

Q 17. Find $\int \frac{2}{(1-x)(1+x^2)}$ dx

Q 18. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

Q 19. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin , then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$

Or

If O be the origin and the coordinates of P be (1, 2, -3), then find the equation of the plane passing through P and perpendicular to OP.

Q 20. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

Or

A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of 1 kg food is given below

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

1 kg of food X costs Rs.16 and one kg of food Y costs Rs.20. Find the least cost of the mixture which will produce the required diet.

Q 21. A card from pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

6× 5 = 30]